Effects of manipulation of perceptual features on children's ability to understand fraction concepts

Research Thesis

Presented in partial fulfillment of the requirements for graduation *with research distinction* in Psychology in the undergraduate colleges of The Ohio State University

by

Madison A. Ramirez

The Ohio State University
May 2018

Author Note

Special thanks to Dr. Tasha Posid and Dr. Vladimir Sloutsky for their collaboration on this research project. I would also like to thank the members of my thesis committee: Dr. Vladimir Sloutsky, Dr. Tasha Posid, and Dr. Seth Miller

Abstract

The formation of a strong foundation in early math education can enable a child to develop a deeper understanding of math in their future academic endeavors. Complex math concepts (e.g., fractions) are particularly challenging for elementary-age children to grasp (Bailey et al., 2012; Department of Education, 1997). In the current study, we explored children's ability to learn and generalize difficult fraction concepts through an abstraction task in the face of varying perceptual information (as encountered in the real world). We presented four-to six-year-old children with a computer task that displayed exemplars of novel fractions that progressed from perceptually impoverished to perceptually rich (concreteness fading; per Fyfe et al., 2014). Through the use of a pre- to post-test design, the present study examined whether children could learn and generalize a novel fraction concept following training (pre- to post-test gains). Data from Experiment 1 show that children could learn a novel fraction concept through this concreteness training in an abstraction task, both for a trained and an untrained exemplar. A follow-up experiment explored the mechanism behind children's success in the original abstraction task. Data from Experiment 2 suggest that children used of a visual strategy when identifying a trained exemplar, but appeared to use a whole number strategy when presented with untrained fractions, such that they succeeded at identifying a new fraction if they could successfully identify its numerator. Pre- to post-test gains on a more traditional fraction task are also discussed. These findings have implications for curriculum development and teacher training.

Keywords: math concepts, abstraction, learning, fractions, perceptual features

Effects of manipulation of perceptual features on children's ability to understand fraction concepts

Children's exposure to basic math concepts begins at an early age. Math is one of the most essential aspects of everyday life. Without a clear and comprehensive introduction to math during children's early development and schooling, it can be difficult for children to fully grasp more complex mathematical concepts and apply them in the future. For example, an understanding of fractions can influence the development of other math concepts, including probability, proportional reasoning, algebra, and much of the STEM fields (Bailey, Hoard, Nugent, & Geary, 2012; Department of Education, 1997). Additionally, a meta-analysis of large datasets of 5th grade children's knowledge of fractions within the U.S. and the U.K. demonstrated that the level of proficiency with fractions was a predictor of general mathematic achievement in the 10th grade (Siegler et al., 2012). Fraction competency is not only attributed to future mathematic achievement, but also other subject areas including biology, physics, chemistry, economics, engineering, sociology, and psychology, and are essential to many common jobs. Thus, a student's level of fraction proficiency can influence his or her ability to succeed in various occupations (Lortie-Forgues, Tian & Siegler, 2015).

Despite the importance of early math understanding and basic math knowledge, elementary and middle school age children in the United States do not demonstrate proficiency in the age-appropriate math skills critical for future math achievement (NMAP, 2008). According to the National Assessment of Educational Progress (NAEP, 1997), only 42% of fourth graders tested could select a picture that represented a visual fraction equivalent to a fraction symbol and only 18% could shade a rectangular region to create an illustration of a given fraction. This limited ability to make connections among the symbolic and visual depictions of fractions indicated that the fourth-graders in this study had a superficial understanding of this fundamental fraction concept (Kouba, Zawojewski, & Strutchens, 1997). The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and the Principles and Standard for School Mathematics (NCTM, 2000) suggest that examining mathematical relationships and building initial concepts of order and equivalence (for example, between whole numbers and fractions) in the elementary grades, would reduce the amount of time teachers would need to spend on addressing these procedural difficulties with older students, thus, resulting in more students succeeding in math.

Besides for a general need to enhance teaching practices and techniques for more difficult math concepts in the classroom (such as fractions and proportions), teachers must also consider children's motivation for learning these difficult and novel concepts. In many cases, teachers rely on the use of concrete, perceptually rich education materials in order to increase children's engagement and retain their interest in the subject matter (Peterson & McNeil, 2012). Although perceptually rich objects may attract attention and increase a child's interest in a task, they can also distract the child from the important information being presented or taught. Particularly, children who have not yet fully mastered the ability to focus their attention and filter irrelevant information are at a disadvantage when presented with extraneous details in the context of learning already-difficult concepts (e.g., math, fractions…). The acquisition of complex relational knowledge (e.g. basic arithmetic, which is foundational in early math and fraction understanding) occurs with development and improvements in relational reasoning (recognizing and applying learned connections to new contexts) are dependent upon increases in cognitive learning and in working memory capacity (Kaminiski, Sloutsky, & Heckler, 2008; Plebanek & Sloutsky, 2017).

Moreover, an individual's ability to successfully recognize and transfer learned relations may be contingent upon the format of the learning material. Some research to date suggests that the use of extraneous features can be detrimental to preschool and elementary school children's learning and transfer of math and relational concepts due to their undeveloped ability to filter out irrelevant information (Kaminski & Sloutsky, 2009, 2013, 2014; Kaminski et al., 2009; McNeil & Fyfe, 2012; Peterson & McNeil, 2012; Plebanek & Sloutsky, 2017; Posid & Cordes, 2014). This poses the question of whether or not the benefits of increased child engagement in a task outweigh the potential negative consequences of the perceptually rich (yet irrelevant) details interfering with the child's overall understanding (Kaminiski et al., 2008; Peterson & McNeil, 2012). For example, in a study by Peterson and McNeil (2012), the authors investigate how perceptually rich objects (e.g., a realistically colored plastic animal or food) might hinder the ability for two- to four-year old children to learn a task when the children already had an understanding of the objects in a non-school setting. Children were randomly assigned to one of four object conditions that varied in perceptual richness (high/low) and established knowledge (high/low). They were asked to watch as a puppet counted an arrangement of objects and were then instructed to indicate whether or not the puppet counted correctly. The results indicated that the perceptually rich objects assisted in the children's counting performance only when they had low knowledge of the objects and interfered in the children's counting performance when they had a high level of prior knowledge. These findings support the authors' hypothesis that prior established knowledge of a perceptually rich object will obstruct the children's learning due to the activation of the objects prior known meaning (Peterson & McNeil, 2012). Therefore, the use of perceptually rich objects may make it more difficult for children to ignore irrelevant features when there is prior established knowledge present.

Additional research on the use of perceptually rich representations and their impact on children's learning and engagement reveal that perceptually rich materials can reduce children's focus and take away from the concepts being taught. For example, Kaminski and Sloutsky (2013) suggest that perceptually rich material can be distracting and a less effective way of teaching children how to interpret bar graphs. In their study, the authors manipulated whether bar graphs contained extraneous features (e.g., flowers or basketballs) or abstract monochromatic bars (e.g., solid colored black). They discovered that six- to eight year old children that learned graph reading with the extraneous features were less accurate interpreting the graphs then the children who were trained to read the monochromatic graphs. Thus, they suggest that the concrete and perceptually rich stimuli were more overwhelming to the learner compared to the more abstract stimuli, thereby interfering with their learning and overall understanding of the concept being taught (Kaminski & Sloutsky, 2013). Additionally, one's ability to comprehend more complex math concepts relies on children's ability to abstract numerical properties despite the presence of extraneous features. Posid and Cordes (2014) investigated three- to six-year-old children's abilities to abstract numerical properties appropriately by asking the children to identify which of two arrangements (perceptually homogenous or heterogeneous) of animals contained a specific number of animals (e.g., "twelve animals") and found that children were more accurate in determining the target number when the animals within the sets were perceptually homogenous. Thus, the authors suggest that the children's greater success with the perceptually homogenous sets was due to the lack of extraneous features distracting the children from the relevant numerical information (Posid & Cordes, 2014). Based on these findings, enhancing one's ability to abstract numerical properties may facilitate a more comprehensive understanding of math concepts and, in turn, aid in one's future math achievements.

Although research suggests that extraneous perceptual information may detract from children's ability to learn about or extract mathematical information from their environment, these vibrant and perceptually rich materials nonetheless are used in the real world, from classroom decorations and materials, to learning apps on computers and tablets, and even in children's educational board games and television shows. Thus, examining how to learn within the context of variable perceptual attributes is warranted. Evidence suggests that children's ability to transfer knowledge from concrete materials effectively is contingent upon the learning environment and guidance presented by the instructor (Brown, McNeil & Glenberg, 2009; McNeil & Uttal, 2009). In an optimally structured learning environment, physical manipulatives (such as solid-colored fraction tiles and black disk counters) facilitate children's understanding and problem solving skills following a presentation of newly learned concepts (Martin & Schwartz, 2005). Moreover, with an effective level of structure, teachers can properly teach children through the use of concrete materials without imposing restrictions on children's freedom to investigate and employ the critical thinking skills that aid in the understanding of new knowledge (Brown et al., 2009). McNeil and Uttal's perspective also supports this notion that teaching methods affect children's learning environment and suggests that the format in which mathematical concepts are introduced to children can have a significant impact on children's ability to comprehend that information (McNeil & Uttal, 2009). Although the use of perceptually concrete materials is prevalent in many teachers' classrooms, it is the context in which they utilize these materials that impacts children's learning. Without proper instruction, children may be overwhelmed or distracted by the perceptually rich stimuli, thus impacting their learning and later generalization (Kaminski & Sloutsky, 2009, 2013, 2014; Kaminski et al., 2009; McNeil & Fyfe, 2012; Peterson & McNeil, 2012; Plebanek & Sloutsky, 2017; Posid & Cordes, 2014).

Therefore, by providing appropriately structured learning conditions and explanatory guidance under the perceptually variable environments in which children are known to learn (e.g. vibrant classrooms, perceptually rich materials), teachers can enhance children's ability to successfully construct and transfer deeper understandings of math concepts.

There is little research specifically investigating *fraction* learning in children and the specific impact of perceptual elements on children's developing understanding of these difficult concepts, particularly given that the current Core Curriculum integrates these rich instantiations into children's instruction (Common Core State Standards Initiative, 2011). Kaminski and Sloutsky (2009) aimed to investigate kindergartener's ability to identify common proportions (the foundational aspects of fraction concepts) when presented in representations that varied in their level of concreteness. They define concreteness by the amount of information a representation is able to communicate. By this definition physical objects would be designated "concrete" rather than images of objects because the physical objects provide additional sensory information. Additionally, the more contextualized concepts are considered concrete due to the information that comes with the more familiar concepts (e.g. symbolic math equations are more abstract). In two experiments Kaminski and Sloutsky (2009) investigate the effect of concreteness on the children's ability to understand and apply their knowledge of proportions. In Experiment 1, the children were given training and test phases on common proportions in either the concrete or generic conditions (e.g. cupcakes with sprinkles out of cupcakes without sprinkles or black circles out of black and white circles). Following the training and testing, the children were presented with a transfer task to match common proportions across different representations. In order to determine the effect of concreteness on the children's ability to identify the common proportions Kaminski and Sloutsky (2009) repeated the procedures of

Experiment 1, but this time did not include the training on proportions. Results showed that children in the generic conditions showed more success in abstracting the relational structure and recognizing common proportions than children in the concrete conditions. These results demonstrate that concreteness can have a negative effect on children's ability to learn and transfer relational knowledge. This research on the effect of concreteness on learning and transfer of proportions can have implications for further investigation of the influence of perceptual elements on children's acquisition of later math concepts such as fractions.

A variety of research (Fyfe, McNeil, Son & Goldstone, 2014; Fyfe, McNeil & Borjas, 2015; Goldstone & Son, 2005; Son, Smith & Goldstone, 2011) provides evidence suggesting that the use of <u>both</u> concrete and abstract representations in conjunction can develop the learning and application of new information by providing children with a range of exemplars with which to form their own understanding of a new concept. Particularly, a phenomenon termed "concreteness fading" (Fyfe, McNeil, Son, & Goldstone, 2014) initially exposed children to concrete materials and slowly transitioned through multiple exemplars to more abstract representations. Concreteness fading emphasizes the relationship between the different levels of perceptual richness and develops children's ability to transfer their learning to new situations (Fyfe et al., 2014). Concreteness fading can be useful in teaching children to recognize similarities or differences that can signal what is important to focus on when attempting to decipher new information and implement the concept in future instances. In a study examining children's ability to detect patterns (a central component to early relational and algebraic knowledge; Son et al., 2011), each child was first shown a simple pattern (small square, large square, small square) and then asked to pick another more complex pattern (diamond, circle, diamond or circle, diamond, diamond) that looked the most like the first pattern. Those who were trained to categorize the sets were more successful in detecting and generalizing the pattern depicting alternating shapes compared to the children who were not taught to look for similarities or differences between the patterns. These results demonstrate the beneficial effects of concreteness fading in improving children's overall grasp of math concepts. In a related study, Fyfe, McNeil and Borjas (2015) investigated the impact of concreteness fading on children's learning compared to teaching methods using concrete-only, abstract-only, or reverse fading (the opposite of concreteness fading: abstract to concrete exemplars). In this study, concreteness fading involved a three-step process of transitioning from the use of concrete materials (e.g., puppets sharing stickers) to more abstract representations (e.g., worksheets depicting illustrations of the puppets or symbolic math equivalence problems) of the concept being taught. Children's learning and performance in transferring the math concepts demonstrated that instruction given through progression from concrete to abstract representations (concreteness fading) provided a deeper conceptual understanding for children learning novel and difficult concepts. In a third example, Goldstone and Son (2005) used computer simulations to examine the role of varying perceptual richness on undergraduate's ability to learn a scientific principle. Undergraduates were presented with a computer simulation of the concrete materials (ants and food) that were switched halfway through the simulation to a more abstract representation (green and black dots) and they were asked to equally distribute the two stimuli in each condition. This initial exposure to concrete representations that transitioned to more abstract representations improved the participants' performance in this simulated tasks. Their hypothesis was similar to that of Fyfe, McNeil and Borjas: concreteness fading should provide a more beneficial method of instruction than the concrete-only, abstract-only, and abstract-to-concrete (reverse) simulations. This hypothesis was supported by the higher level of improvement from the participants trained with concreteness fading. Each of these studies provides evidence of improvement in participants' learning after being trained progressively from concrete to abstract representation (concreteness fading). Together, these studies suggest that the combined use of concrete and abstract learning – specifically in a progression that introduces students first to concrete and last to abstract materials - increases their level of understanding of that novel information. This use of concreteness fading for the teaching of novel or difficult concepts is the subject of the current study.

Current Study

The aim of the current study was to investigate whether the presentation of perceptual features of stimuli from concrete to abstract representations (concreteness fading) can facilitate a child's understanding of novel math concepts, specifically fractions, which has been successful in non-fraction studies utilizing a similar paradigm. We also investigated children's ability to generalize the newly-learned fraction concepts both during training and from pre- to post-test.

In two experiments, we investigate (a) whether the use of concreteness fading as a strategy to present novel fraction concepts impacts children's learning and generalization of those concepts. We employ a pre- to post-test design in order to investigate whether (b) children are able to generalize and transfer this new fraction knowledge immediately following training and from pre- to post-test and (c) whether prior math knowledge impacts children's ability to learn novel fraction concepts under perceptual variability. In Experiment 1, we test the use of a known paradigm (simple-to-complex abstraction, or concreteness fading; per Son et al., 2008) in the context of fraction learning. We ask whether children can learn a novel fraction and generalize post-training, and whether this may further extend to non-practiced exemplars. In Experiment 2, we examined the mechanism behind children's performance in Experiment 1 by employing the same abstraction paradigm. However, here we manipulated the label presented to children with visual fractions in order to examine whether their success in Experiment 1 was due to a visual strategy (e.g., the fraction "looks like" the exemplar from training) or a whole number strategy (e.g., there are three pieces colored in, regardless of the total number of pieces).

Experiment 1

Method

Participants

Thirty-five 4-6 year old children participated in this study (M_{age} = 5.26 years, *SD*=1.04 years; *n*=17 male and *n*=18 female). Children were randomly assigned to one of two testing Conditions: (1) Single Exemplar (*n*=20, described below) or Multiple Exemplar (*n*=15, described below). The children were recruited through and tested at the OSU Cognitive Development Lab, at local elementary schools in the Columbus area, or at the Center of Science and Industry (COSI). Data from six children were not included in the statistical analyses because they did not complete all components of the study.

Materials

Pre-Test. Children first completed a fraction-matching task, in which they were presented with a three blocks of match-to-sample questions (see Figure 1). Each block consisted of 12 trials for a total 36 trials. The first and third blocks pictured a sample image of a fraction picture (i.e., black and white circle) and children were asked to match that image to one of four multiple-choice symbolic answers. The second block utilized a symbolic fraction (i.e., 2/3) as the sample image and children were asked to match that symbolic fraction to one of four multiple-choice picture answers (black and white circles).

Abstraction Task. The abstraction task was administered directly following pre-test and consisted of a training phase and a test phase. In both Conditions, during the training phase, children were presented with a single fraction (3/4), which was presented visually and across perceptual variability (see Figure 2). The fraction images increased in perceptual variability across five trial types: (1) black and white circles, (2) colorful circles, (3) black and white squares, (4) colorful squares, and (5) perceptually rich item (pizza pie). Each trial type presented two exemplars, for a total of ten training trials. Children were told, “This is three-fourths! See, three pieces are colored in out of four total pieces!”

Training was followed by the test phase of the abstraction task (Figure 3). The test portion consisted of a total of 24 intermixed test trials representing stimuli ranging from perceptual impoverished to perceptually rich, as per the five trial types in training. The questions on the test portions were reflective of the children’s assigned training condition (Single Exemplar or Multiple Exemplar). In the Single Exemplar Condition, children were always asked to find “three-fourths” and were presented with four multiple-choice options. All answer options were representative of a single trial type (e.g., all black and white circles, or all colorful squares; see Figure 3). That is, stimuli did not vary within each trial but varied across trial types. In the Multiple Exemplar Condition, children were asked to find “three-fourths” for a quarter of the trials, and were asked to find a different, non-trained fraction on the remainder of the trials (e.g., “two-thirds,” “one-eighth”…). The actual answer choices were identical to the Single Exemplar Condition, such that just the fraction which children were asked to find differed, not the choices themselves. Again, the stimuli did not vary within each trial but varied across trial types.

Post-Test. The post-test was completed immediately after the abstraction task and was identical to pre-test.

Each computer task was presented on a 13-inch MacBook laptop and all programming was done using RealBasic software. This software also recorded children's answers and reaction time when completing these tasks.

Procedures

Pre-Test. Children were first asked to complete the pre-test in order to gauge the children's level of prior fraction knowledge. The children completed a fraction-matching task (Figure 1), in which they were presented with three blocks of match-to-sample questions that each consisted of 12 trials. The first block presented a numerical fraction (e.g., 4/6) in the top center of the screen and children were asked to pick the shape on the bottom (from four multiple-choice options) that best represented the number. In the second block, children were presented with the opposite: they were asked to pick the symbolic fraction that best matched the visually presented sample fraction. In the third block, children were again presented with a symbolic fraction and then were asked to match it to a corresponding visual depiction (e.g., a circle with 4 of 6 shaded slices).

Abstraction Task: Training was identical across the Single Exemplar and Multiple Exemplar Conditions. Children were taught that, no matter the change in perceptual features or orientation (e.g., three-fourths as a black-and-white circle, three-fourths as a purple-and-black circle, three-fourths as a black-and-white square, three-fourths as a red-and-white circle, three-fourths as a pizza pie; Figure 2), the fraction "3/4" referred to all of these variations. In test, children were presented with questions illustrating the various degrees of perceptual richness they were trained on and asked to match the correct visual fraction to the fraction term in question. The fraction they were tested on was dependent on whether they were part of the Single or Multiple Exemplar group (e.g. Single Exemplar tested only on the fraction from training (3/4) and Multiple Exemplar tested asked about both 3/4 and other untrained fractions).

The test phase consisted of twenty-four questions that only differed in the fraction they were asked to identify.

Post-Test. Immediately following the abstraction task, children completed a fraction matching task identical to pre-test. The changes in performance (difference score) between pre- and post-test were used to indicate children's generalization from the abstraction training task.

Results and discussion

The current study examined four outcome variables of interest: (1) Learning (accuracy) in the abstraction task: Did the children perform above chance (25%) on the test trials of the abstraction task following the training? (2) Trial type differences: Did the children show better or worse performance on certain trial types? That is, did we see any difference in accuracy across trial types in the abstraction task? (3) Pre-post test gains: Did the children show improvement from the pre-test to the post-test? (4) Predictors of learning: What factors predicted learning in the abstraction task?

Single Exemplar (Experiment 1A):

The results from the Single Exemplar condition indicate that children were able to learn during the abstraction task due to their above-chance performance (vs. 25%; M=76.6%, $t(19)=8.03$, $p<.001$, *Cohen's d*=3.68; Figure 4). This held across all trial types (all *p's*<.001, *Cohen's d's*>2.5; see Table 1 and Figure 4). There was no difference in accuracy by trial type ($F(4,76)=2.23$, $p=.073$, $\eta_p^2=.105$).

We also examined whether participants improved from pre- to post-test on our more traditional fraction-matching task. We calculated a difference score (post-test accuracy minus pre-test accuracy) for each participant. The average difference score for participants in the Single Exemplar condition was not significantly different from zero, indicating no pre- to post-test gains (*p*>.7; Table 2 and Figure 5). Finally, we ran a linear regression model to examine what factors predicted children's accuracy in the abstraction task. Neither independent variables of interest (age or accuracy on the pre-test task) significantly predicted accuracy on the abstraction task (*p'*s>.1; Model: R^2=.181, *p*=.183).

Multiple Exemplar (Experiment 1B):

For the Multiple Exemplar data, we looked at children's accuracy on both the fraction on which they were trained (3/4), as well as their performance on the other non-trained (i.e., not 3/4) fractions. As in the Single Exemplar condition, when asked to identify the fraction ¾, children were able to do so and performed significantly above chance (25%; *M*=65.6%, *t*(14)=4.38, *p*=.001, *Cohen's d*=2.34; Figure 4). Again, this generally held across trial types (all *p's*<.053, *Cohen's d's*>1.14; Table 1 and Figure 4).

Critically, children also performed above chance-level (25%) when asked about non-3/4 fractions (*M*=55.6%, *t*(14)=4.34, *p*=.001, *Cohen's d*=2.32; Figure 4). This also remained consistent across trial types (all *p's*<.05, *Cohen's d's*>1.2; Table 1 and Figure 4) and there was no significant difference across trial types in the abstraction task (*t*(14)=1.99, *p*=.067, *Cohen's d*=1.06). We also compared children's accuracy across trained (3/4) and non-trained (non-3/4) test questions. There was no difference in accuracy on trials that asked about ¾ and trials that did not ask about ¾ (*p*>.05).

Unlike in the Single Exemplar condition, pre to post-test gains were observed (*t*(14)=2.36, *p*=.033, *Cohen's d*=1.26; Table 2 and Figure 5). Due to the presence of pre- to post-test gains, we further investigated whether a specific portion of the Matching Task (i.e. a certain block of trials) showed the most improvement from pre- to post-test. We find that children showed improvement when they were presented with a sample symbol and were asked to match the symbol to a corresponding picture ($t(14)=3.64$, $p=.003$, *Cohen's d*=1.95). The other fraction task sections (p's>.4) did not show improvement.

Again, we examined predictors of children's learning in the abstraction task through a linear regression model. Regression analyses demonstrated that age was a significant predictor ($\beta=.869$, $p<.001$) of performance in the abstraction task, but pretest accuracy was not ($p>.7$; Model: $R^2=.835$, $p<.001$).

In Experiment 1, we examined whether the use of concreteness fading as a strategy to present varying perceptual features could aid in children's ability to learn and generalize novel fraction concepts. Results indicate that children could learn a novel fraction concept through this concreteness training in our abstraction task. For the Single Exemplar condition, children did not show any gains on the pre- to post-test fraction task. Results from the Multiple Exemplar condition further demonstrated that they could generalize this new abstraction ability to identify non-trained fractions. Additionally, there were pre- to post-test gains on a more traditional measure of fraction knowledge following the Multiple Exemplar condition. Although the data in Experiment 1 show that children were able to succeed in the abstraction task, it did not provide insight into what mechanism the children were relying on in order to succeed. This question was addressed in Experiment 2.

Experiment 2

The results of Experiment 1 demonstrate that children are able to learn a novel fraction concept through concreteness fading in an abstraction task. Although the results of Experiment 1 showed a difference in the children's pre- to post-test gains based on whether they were placed in the Single Exemplar or Multiple Exemplar conditions, it did not give an explanation as to <u>why</u> there was a difference. Accordingly, Experiment 2 investigated the possible mechanism behind children's success in the abstraction task in Experiment 1, with the possibility that this mechanism might be different across the two conditions.

In Experiment 2, we presented children with one of two conditions. We explore the use of a visual strategy (Visual Label Strategy; e.g., the fraction "looks like" the exemplar from training) or a whole number strategy (that is, a focus on the numerator only; e.g., there are three pieces colored in, regardless of the total number of pieces).

Visual Label Strategy. An object category can be created when designating the same name to a set of different objects, which as a result highlights similarities among the objects. For example, by using the same label across four different animals, infants as young as 12 months of age can form an object category such as "animal" (Waxman & Markow, 1995; also see Ferry, Hespos, & Waxan, 2010; Fulkerson & Waxman, 2007). Geraghty, Waxman, & Gelman (2014) conducted a study to examine whether infants could interpret a novel word as an object category, testing this visual-label hypothesis. Specifically, 15- and 17-months old infants were presented with a picture of a novel noun and told it's name (for example, a whisk) to see if they could only extend the novel noun to other pictures exactly the same as the stimuli presented, to pictures just differing in color, or to representations that differ in both color and representational medium (a three-dimensional silver whisk). They found that children did not show a strong reliance on the perceptual similarity or a preference for the physical objects rather than the pictures. Instead, children interpreted the novel noun to an object concept, meaning the word was enough to associate to new members of the same object category even when they differed in color and representational modality (Geraghty et al., 2014). This poses the question of whether children's high success in the Single Exemplar condition of Experiment 1 was a result of their ability to form object category after being taught <u>any</u> label, not necessarily "three-fourths". Thus, Experiment 2A utilizes the same paradigm as in the Single Exemplar condition, but rather than associate the visual fractions with the term “three-fourths,” we used a nonsense label, “dax.”

Whole Number Strategy. Research to date demonstrates that many people – and especially children – demonstrate a “whole number bias” when thinking about fractions. Children often default to whole number counting strategies, for example, when countable units are available (Boyer et al., 2008; Kaminski & Sloutsky, 2013), which is unsurprising give that children are first exposed to whole numbers before being introduced to fractions (made up of two whole numbers) in the Common Core (Common Core Standards Initiative, 2011). However, research in mathematics education also suggests that the use of children’s intuitive knowledge to scaffold their learning of new concepts may be beneficial (Boyer & Levine, 2012; Halberda, Mazzocco, & Feigenson, 2008). Because children may not understand that the numbers in a fraction’s numerator and denominator stand for counts (thus making up a part-whole relationship), perhaps they were simply relying on the numerator to complete the abstraction task following training. Thus Experiment 2B utilizes the same paradigm as in the Single Exemplar condition, but rather than use a fraction term (“three-fourths”), we simply used the numerator (“three”) to describe the novel fractions during training.

Method

Participants

Forty-one 4-6 year old children participated in Experiment 2 (*Mage* = 4.88 years, *SD*=0.87 years; *n*=19 male and *n*=22 female). Children were randomly assigned to one of two conditions: Dax Exemplar (n=20, described below) or Three Exemplar (n=21, described below). The children were recruited through and tested at the OSU Cognitive Development Lab, at local elementary schools in the Columbus area, or at the Center of Science and Industry (COSI). Data from two children were not included due to failure to complete all components of the experiment.

Materials

Pre-Test. The fraction-matching task was identical to that of Experiment 1.

Abstraction Task. The abstraction task was identical to that of Experiment 1, with the following differences: In the Dax Exemplar[1] condition, children saw training images that were identical in make-up and sequence as those presented in Experiment 1. However, children heard the following description, "This is dax! See, three pieces are colored in out of four total pieces!" Similarly, in the Three Exemplar condition, children saw training images that were identical to those in Experiment 1; however, children heard the following description, "This is three! See, three pieces are colored in out of four pieces total!"

Identical to the Single Exemplar condition of Experiment 1, the test portion of the abstraction task consisted of a total of 24 test trials representing stimuli ranging from perceptual impoverished to perceptually rich. Children were asked to select "dax" in the Dax Exemplar condition or "three" in the Three Exemplar condition for all test questions. The four multiple-choice answer options were identical to Experiment 1 and children's answers were indicative of a three-fourths selection, such that the correct answer was also identical to the Single Exemplar condition of Experiment 1.

Post-Test. The post-test was completed after the abstraction task and was identical to the post-test used in Experiment 1.

Each computer task was presented on a 13-inch MacBook laptop and all programming

---

[1] Many studies to date have used "nonsense" words that do not exist in the English language, yet contain the same structure as words that currently do exist (for other examples, see Deng & Sloutsky, 2012, 2013, 2015). These novel labels are labels with no known reference and have been used to investigate the mapping of novel objects and categories (e.g., Ferry et al., 2010; Fulkerson & Waxman, 2007; Halberda, 2003).

was done using RealBasic software. This software also recorded children's answers and reaction time when completing these tasks.

Procedures

The procedure of Experiment 2 was identical to that of Experiment 1, with the children participating in the pre-test, abstraction task, and post-test. The pre-test and post-test were identical to that of Experiment 1. The procedure used in Experiment 2 were identical to Experiment 1, except that children in the Dax Exemplar condition heard the label "Dax" throughout training and test and the children in the Three Exemplar condition heard the label "Three" throughout training and test.

Results and discussion

The purpose of Experiment 2 was to collect follow-up data to investigate the mechanism for children's success in the abstraction task and condition variability in their pre- to post-test gains in Experiment 1. Again, the outcome variables of interest were as follows: (1) Learning (accuracy) in the abstraction task:: Did the children perform above chance (25%) on the test trials of the abstraction task following the training? (2) Trial type differences: Did the children show better or worse performance on certain trial types? That is, did we see any difference in accuracy across trial types in the abstraction task? (3) Pre-post test gains: Did the children show improvement from the pre-test to the post-test? (4) Predictors of learning: What factors predicted learning in the abstraction task? For Experiment 2, we additionally asked: (5) Were there differences across the experiments in terms of accuracy, gains, and/or predictors of learning? That is, did performance (either accuracy or gains) in the Dax Exemplar or Three Exemplar conditions mirror performance in the Single Exemplar or Multiple Exemplar tasks? This would be suggestive of similar strategies utilized by participants across these conditions.

Dax Exemplar (Experiment 2A):

As in Experiment 1, we first asked whether children performed above chance-level (25%) on the test trials of the abstraction task. The results indicate that children performed above chance overall on the abstraction task (M=79.6%, $t(18)=9.15$, $p<.001$, *Cohen's d*=4.31; Figure 6) and across the five trial types (all p*'s*<.001, *Cohen's d's*>3.11; Table 1 and Figure 6). Results did reveal a slight difference in accuracy by trial type ($F(4,72)=3.32$, $p=.015$, $\eta_p^2=.156$), with children performing slightly less accurately on the black and white non-circles trial type. This was consistent with results from other conditions.

We also assessed any pre- to post-test gains on our fraction matching task. The average difference score was not significantly different from zero, indicating no pre to post-test gains ($p>.4$) in the Dax Exemplar condition (Table 2 and Figure 5). Next, we assessed predictors of children's accuracy on the abstraction task using a linear regression. Age ($\beta=.625$, $p=.009$), but not pre-test accuracy ($p>.9$), was predictive of performance on the abstraction task (Model: $R^2=.391$, $p=.019$).

Finally, we compared accuracy in the abstraction task and pre- to post-test gains on the matching task across the Single Exemplar and Dax Exemplar conditions. The accuracy on the abstraction task ($p>.7$) did not differ across these conditions (Dax: M=79.6% vs. Single: M=76.7%). This held across the five trial types as well (all p's>.5). Participants did not demonstrate any pre- to post-test gains in either the Single or Dax Exemplar conditions, and the mean difference score for each of these conditions did not differ ($p>.5$). These comparison results indicate that children in the Single and Dax Exemplar conditions may have been utilizing the same strategy (a visual, general label strategy) to complete both of these tasks.

Three Exemplar (Experiment 2B):

We first examined whether children performed above chance-level (25%) on the abstraction task. Children performed significantly above chance overall on the abstraction task (*M*=84.5%, $t(20)=14.9$, $p<.001$, *Cohen's d*=6.7; Figure 6) and across the five trial types (all *p's*<.001, *Cohen's d's*>3.2, Table 1 and Figure 6). As in Experiment 2A, there was a slight difference in accuracy by trial type ($F(1,20)=10.01$, $p<.001$ $\eta_p^2=.335$) because children were slightly less accurate on the black and white non-circles trial type, consistent with other conditions.

Pre- to post-test gains were present following training with the Three Exemplar ($t(20)=3.8$, $p=.001$, *Cohen's d*=1.7; Table 2) and the average difference score was positive (*M*=10.3%, $t(20)=2.8$, $p=.013$, *Cohen's d*=1.2; Figure 5). Next, we assessed predictors of learning in the abstraction task. Neither age nor pre-test accuracy significantly predicted accuracy on the abstraction task (*p'*s>.3; Model: $R^2=.391$, $p=.019$).

Next, we compared performance on the abstraction task and pre- to post-test gains to the corresponding results from the Single Exemplar condition. Although accuracy between the two tasks did not statistically differ ($p>.3$; across trials types: all *p'*s>.1), there was a difference between pre- to post-test gains (M_{gains}=.97% vs *M*=10.3%; $t(39)=1.84$, $p=.073$, *Cohen's d*=.59). This difference in pre- to post-test gains across the Single and Three Exemplar conditions suggests that children were not initially relying on this whole number strategy in the Single Exemplar condition.

We then compared performance on the abstraction task and pre- to post-test gains to that from the Multiple Exemplar condition. Although the overall accuracy score differed ($t(34)=2.08$, $p=.045$, *Cohen's d*=0.71) between the conditions, a break-down by trial type revealed that only one trial type actually significantly differed (black and white non-circles; $t(34)=2.38$, $p=.023$, *Cohen's d*=0.82). With the exclusion of this one trial type , average accuracy on the abstraction task did not differ across the Multiple Exemplar and Three Exemplar conditions (p's>.09, *Cohen's d'*s<.6). We also examined the significant pre- to post-test gains demonstrated in both the Multiple and Three Exemplar conditions. There was no statistical difference between Gains in the Three Exemplar (M=10.3%) and the Multiple Exemplar (M=8.3%), suggesting that children in the Three condition were successful in the Multiple Exemplar condition because they were using a whole number strategy (that is, using the numerator of the presented fraction rather than the part-whole relationship). The slight difference in accuracy across the abstraction task may indicate that, whereas children in the Multiple Exemplar condition had to figure out a strategy on their own (and subsequently used a whole number strategy), they did not have to go through this "learning" experience in the Three Exemplar condition, having been given this whole number strategy instead.

Cross-Experiment Analyses:

We ran a final series of cross-experiment analyses to determine if there were any variables predictive of accuracy on the abstraction task or pre-post-test gains, specifically by testing Condition.

We first ran a linear regression, which included data from all four of our training conditions. We included Age, Condition, and Pre-Test Accuracy as predictors in our model and our dependent variable was children's accuracy on the abstraction task. Age (β=.447, p=.001) and condition (β=.277, p=.008) were predictive of children's accuracy on the abstraction task, but pre-test accuracy (p>.3) was not significant (Model: R^2=.285, p=.001). Follow-up analyses reveal that children do worse on the abstraction task in the Multiple Condition (p's<.04). The other three conditions did not differ from one another (all p's>.3).

We ran a second linear regression with same predictors entered as independent variables. Our dependent variable was pre- to post-test gains on our fraction matching task. Neither age, pre-test accuracy, nor condition were predictive of pre-post-test gain (all *p'*s<.04; Model: R^2=.408, *p*>.3), most likely due to the fact that we did not see a high percentage of gains overall in the task. However, we do see a significant difference in Gains between Single/Dax Exemplars (collapsed) and Multiple/Three Exemplars (collapsed; *p*<.05).

Overall, children showed success in learning across both conditions in Experiment 2, as observed across Experiment 1. The accuracy in the abstraction task and lack of gains from pre- to post-test observed between the Dax Exemplar and Single Exemplar conditions are very similar, indicating the children were likely looking at the overall visual shape of the representations in both of these conditions. The similarity in the children's performance in the Multiple Exemplar and Three Exemplar conditions suggests the children were relying on the use of the whole number strategy. The implications of these findings are further discussed in the General Discussion.

General Discussion

Math is a skill used in everyday life and it is vital that children be presented with a clear and comprehensive introduction to math at an early age of development. Without a strong foundation in early math education it can be difficult for children to fully understand more complex math concepts (e.g., fractions and proportions) and apply them in the future (Bailey et al., 2012; Department of Education, 1997). It is common for teachers to rely on the use of concrete education materials with the goal of increasing children's engagement within the classroom (Peterson & McNeil, 2012). However, the use of the extraneous education materials can be challenging for younger children to fully learn and transfer math concepts due to the undeveloped ability to ignore irrelevant information (Kaminski & Sloutsky, 2009, 2013, 2014; Kaminski et al., 2009; McNeil & Fyfe, 2012; Peterson & McNeil, 2012; Plebanek & Sloutsky, 2017; Posid & Cordes, 2014). In order to overcome the potential negative consequences associated with instructors only relying on either concrete or abstract representations, some research suggests that it may be beneficial to use both concrete and abstract representations together to promote learning through multiple exemplars of new concepts ("concreteness fading;" Fyfe et al., 2014; Fyfe & McNeil 2009; Son et al., 2008). Although there has been research on the use of concreteness fading as an instructional tool, this study is the first to examine this potentially beneficial teaching technique in fraction training. Accordingly, in two experiments, the present study investigated how the use of concreteness fading as a strategy to expose children to novel fraction concepts in the face of varying perceptual features as a way to promote learning and generalization of those novel concepts.

Experiment 1 investigated whether the use of concreteness fading as a strategy to present novel fraction concepts can aid in the children's ability to learn and generalize these concepts prior to formal education. Results indicated that children were successful in the abstraction paradigm across both the Single Exemplar and Multiple Exemplar conditions, as well as for both trained and untrained fractions. We can conclude that the use of the concreteness fading strategy did prove to be an effective way for children to understand novel fraction concepts, which has not been previously demonstrated.

There has been some previous research on the effectiveness of concreteness fading to support findings in this study. Son et al. (2008) examined whether presenting toddlers with either simple or complex training exemplars, through the use of novel object categories, could condense the process of abstraction and stimulate the process of generalization. The children were taught an unfamiliar name, either to a perceptually rich complex or a simplified geometric shape, and then tested to see how likely they were to generalize based on shape. They found that children were more successful in generalizing according to shape similarity when they were trained with simple objects rather than those who were trained with complex objects (Son et al., 2008). These results not only supported that abstraction was a product of learning but demonstrated the important role simplified objects can play in children's rate of learning and generalization compared to the use of only complex exemplars.

It is not surprising that concreteness fading was a successful strategy for teaching children novel fraction concepts in our study, as other work also suggests that the use of a combination of perceptually impoverished and perceptually rich representations may promote the learning of difficult math concepts, such as fractions and proportions, especially when they are first introduced to young children. For example, Posid & Sloutsky (2017) examined whether the instructional format in which children were initially taught about fractions would affect their learning and transfer of these novel fraction concepts. They did so by manipulating the extent to which real-world training was established in visual or symbolic representations. Children who received additional perceptual information (in the Visual+Symbol condition compared to a Visual-Only or Symbols-Only training condition) showed a deeper level of understanding and ability to employ the concepts being taught. These findings suggest that understanding new and difficult fractions – especially in a task requiring generalization and far transfer – may require additional (perceptual) information to help the children develop a mapping of the concept (Posid & Sloutsky, 2017). In another example, Posid & Cordes (2017) demonstrated that presenting multiple pieces of information in conjunction promotes the mapping of novel math concepts, particularly for young children. In this study, when the researchers counted the individual stimuli in addition to highlighting the overall number of the set when presenting it to the children (highlighting the concept of cardinality; Posid & Cordes, 2017), children developed a deeper mapping of the concept being taught (also see: Mix et al., 2012). This previous research reinforces the present study's conclusion of the strategy concreteness fading (a combination of exemplars to induce broader generalization) being an effective method of instruction for children learning novel fraction concepts prior to formal education.

It is also interesting to note that children were successful in the Multiple Exemplar condition even though they did not receive training on the non-3/4 fractions included in the test phase of the abstraction task. Children's success in this condition demonstrated their ability to generalize to new fraction knowledge and make inferences that led them to extend their understandings to non-practiced fractions. Therefore, it may be advantageous for future research to look at possibly training children on multiple fractions during training itself to see if their learning during the test phase of the abstraction task (or in their pre- to post-test gains) increases.

The results in Experiment 1 also revealed pre- to post-test gains in the Multiple Exemplar, but not the Single Exemplar, condition. It is interesting that there were any improvements to be seen with the use of such a short and passive training. Despite this, our short abstraction training helped children to focus their attention on the important aspects of the fraction concepts while ignoring any irrelevant features. By including multiple fraction representations in the Multiple Exemplar condition, children were able to generalize better than in the Single Exemplar condition that only exposed children to the fraction ¾. This was presumably because they had to consider what the term was that they had been presented with (e.g., what does "three-fourths" or "seven-eighths" actually mean), rather than possibly using it as a non-matched label (e.g., in the Single Exemplar condition, children may not have actually understood that "three-fourths" represented a part-whole relationship; that is, maybe we could have said “pacman” and they would have also succeeded). It was this difference and individual variability in these conditions that lead us to look at the cause of this disparity and children’s mechanism for success in Experiment 2.

It should be noted that gains in our study did not exceed 10% overall and no gains were present in the Single Exemplar condition. Additionally, we saw generally small effect sizes in terms of gains. This brings into question whether such gains may be experiment-specific or whether this would translate to the general population. This could be addressed in one of two ways: (1) Through the use of a control condition or (b) through the use of (or inclusion in) a meta-analysis. A control condition would be useful in determining whether or not it was the use of the concreteness fading paradigm per se – and specifically the training – that was effective in teaching children the novel fractions, rather than just exposure to the task or simple practice effects. For example, if we did not train children on concreteness fading would the children show the same high accuracy in the abstraction task or in their pre- to post-test gains. I hypothesize that they would not be as successful without the concreteness fading strategy because they would not be exposed to multiple exemplars – the hallmark of concreteness fading – and thus there would be less emphasis placed on the important aspects of the novel fraction representations needed for the children to fully grasp the numerical concept being taught. In fact, not only might this detract from learning the numerical information, but it could also emphasize non-important (non-numerical) perceptual features on which to focus (e.g., see Deng & Sloutsky, 2015; Posid, Mills, & Sloutsky, in preparation). Due to the low number of participants in this single study (i.e., 20 or so participants per condition) we cannot say with certainty that it could be replicated or that, practically-speaking, this paradigm should be immediately implemented into curriculum or teaching training. However, a meta-analysis and comparison of other studies findings may be beneficial in generalizing our findings when communicating to teachers the effective strategies to take into consideration when presenting math concepts.

The questions raised about the mechanism behind children's success in Experiment 1 were addressed by running Experiment 2. The Dax Exemplar condition was employed to test whether the children were using a visual (label) strategy in the Single Exemplar condition. Previous research indicates that by providing a label in conjunction with a concept being presented, it can help children understand the mapping between the label and the item (Ferry et al., 2010; Fulkerson & Waxman, 2007; Waxman & Braun, 2005; Waxman & Markow, 1995). Conversely, the Three Exemplar condition was implemented in order to test if the children were relying on a whole-number-strategy in Experiment 1. Due to children's tendency to show a whole-number bias when completing fraction and proportion tasks (Boyer et al., 2008; Kaminski & Sloutsky, 2013), children in our study may have been relying on their prior knowledge of whole numbers when interpreting the novel fraction concepts. This would lead the children to focus on the numerator of the fractions being presented and not taking into count the denominator following the abstraction task.

The similarities of the results in the Single Exemplar and Dax Exemplar conditions suggest that children in the Single Exemplar condition of Experiment 1 may have been relying on a visual labeling strategy (Ferry et al., 2010; Fulkerson & Waxman, 2007; Waxman & Braun, 2005; Waxman & Markow, 1995). Children may have been focusing on the specific shape of the shaded-in portions of the ¾ image representations from the training exemplars and using this overall shape bias (see Cantrell & Smith, 2013, for a broader discussion) when choosing their answers. There were also no apparent gains across the Single Exemplar and Dax Exemplar conditions, which suggests that rather than looking at the symbolic fractions (or paying attention to the actual label being used), children were only looking at the relative holistic “shape” of the exemplar. Thus, the children’s focus on the shape was what helped children succeed in the task, but did not result in their ability to generalize that knowledge in a more traditional fraction task at post-test.

The Multiple Exemplar and the Three Exemplar conditions also resulted in similar patterns of performance. A fraction is just two whole numbers, and young children often have trouble transitioning from their use of whole numbers to the use of whole numbers as a fraction (with a part-whole relationship), as currently presented in the Core Curriculum (Common Core Standard Initiative, 2011). Children in the four-to-six age range have yet to learn fractions, so results from the present study suggest that they attempt to use knowledge they already have (that is, about whole numbers) and apply it to the concept at hand (that is, fractions), which children have also shown to do in other studies (Hurst & Cordes 2016; Lewis, Mathews & Hubbard, 2015; Ni & Zhou, 2005; Siegler et al., 2013). Both conditions also showed pre- to post-test gains; however, these gains were quite small (only ~10%). This suggests that children’s whole number strategy was more helpful in the abstraction task, with easier, countable units, than on a more traditional fraction-matching task on which they were not trained. For example, the nature of pre- to post-test did not allow for children to match numerators-only because the options included multiples of the same numerators (for example, 3/4 and 3/8 were both answer choices) making it impossible for children to only rely on the whole numbers only.

Children showed more success in the Three Exemplar condition than the children assigned to the Dax Exemplar condition. These exemplars are representative of the two different strategies the children could have been relying upon to succeed in Experiment 1. The children in the Three Exemplar condition may have shown more success because the test phase did not present multiple fraction answer choices with the same numerator (e.g., three-fifths). Once the children were trained on this whole number strategy, all they really had to look for were the three colored-in pieces of the representations (the numerator). This is not a strategy that would allow children to be completely successful in learning fractions, but could allow them to answer correctly when the answer choices did not present multiple same-numerator options. It may be developmentally interesting to run a study in which the Three Exemplar and Dax Exemplar are pitted against one another to see if children are using one strategy over the other (either preferentially or developmentally). For example, in this proposed study, the children could be asked to find three-fourths but not have that as an answer choice. Instead, they would be presented with, for example, six-eights (visually identical to three-fourths) or three-fifths (same number of pieces colored in, representing a correct numerator match) to see whether they choose based on the visual shape or by the numerator. However, even if you did pit Three and Dax against one another, its important to note that these strategies alone would not be useful in teaching children the fraction concepts because they do not fully address the holistic part-whole relationship of a fraction. Thus, although we suggest concreteness fading as a method to teach children about a novel math concept, such as fractions, this suggestion should be further researched to develop a paradigm that successfully highlights a part-whole relationship.

The present study had a few limitations. As previously mentioned, there was no control condition to provide us with a baseline account of accuracy on the abstraction ask or for children’s pre- to post-test gains. Although children’s above-chance accuracy on the abstraction task across four testing conditions and multiple testing locations (in lab, at schools, at a local children’s museum) suggest these results would replicate, a baseline of comparison is warranted.

Additionally, the short three-minute training intervention we utilized was a passive viewing paradigm that did not allow any observations of lasting effects like an active and/or longer training paradigm may have produced. The children tested were also much younger than are children who are beginning to learn fractions in school (3rd or 4th grade, per Common Core Standard Initiative, 2011). It would be interesting to examine whether the same results would be observed if the participants were actively learning fraction concepts. Would this paradigm be just as effective? Or might the paradigm be more effective because children would presumably have better context and knowledge so as to avoid strategies like whole number strategies? Future work should examine this.

One additional limitation is that this study does not speak to the underlying cognitive mechanism(s) that lead to children's success in this task. For example, we did not measure children's attention, working memory capacity, or inhibitory control. Future work could either measure these as predictive variables of learning and performance on our task or we could employ eye-tracking technology to look at these types of factors while children are completing this task. We speculate that children's attentional control will be an important factor in their learning and generalization in our task, which in turn should impact their ability to hold this new information in memory. Support for this theory comes from work within our lab suggesting that children distribute their attention to both relevant and non-relevant information when learning novel concepts (Deng & Sloutsky, 2015, 2016). Similarly, other work suggests selective attention is particularly important when learning under difficult tasks demands, such that greater task demands lower working learning memory capacity both in children and adults (Plebanek & Sloutsky, 2017; Posid et. al., in preparation). In sum, future work should examine the cognitive mechanism(s) that lead to children's success.

Possible future research ventures could investigate the use of the other progressions coined by Son et al. (2008), such as a similar complex-to-simple training. Additionally, it would be interesting to see if training on multiple fractions would lead to better abstraction and pre- to post-test gains, following our observation that children demonstrated more gains following the Multiple Exemplar training as compared to the Single Exemplar training.

In conclusion, results from the present study suggest that pre-school age children can successfully learn about fractions prior to formal education through the use of a concreteness fading strategy in an abstraction task. Even with such a short and passive training paradigm, children were able to learn and generalize this new knowledge to both familiar and unfamiliar fractions in the face of perceptual variability. Results from Experiment 2 suggest that children employed different strategies across Experiment 1: when asked to find a single fraction on which they were trained, children likely used a visual label strategy. However, when asked to find multiple fractions (trained and untrained), children likely used a whole number strategy. This is important given the evidence attributing fractions to being predictive of future math achievement as well as being valuable in other subject areas that are crucial for success in various occupations (Lortie-Forgues, Tian & Siegler, 2015). Therefore, the current study emphasizes the usefulness of finding new educational methods that expose children to complex math concepts prior to formal education in the classroom and how these methods can serve as a basis that enables children to develop a stronger understanding of math in their future academic endeavors.

References

Bailey, D.H., Hoard, M.K., Nugent, L., & Geary, D.C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology, 113*, 447-455.

Boyer, T. W., & Levine, S. C. (2012). Child proportional scaling: Is 1/3= 2/6= 3/9=4/12?.*Journal of Experimental Child Psychology*, 111(3), 516-533.

Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: where young children go wrong. *Developmental psychology, 44*(5), 1478.

Brown, McNeil, and Glenberg (2009). Using concreteness in education: Real problems, potential solutions. Child Development Perspectives, 3(3), 160-164.

Deng, W., & Sloutsky, V. M. (2012). Carrot eaters or moving heads: Inductive inference is better supported by salient features than by category labels. Psychological Science, 23, 178–186.

Deng, W., & Sloutsky, V. M. (2013). The role of linguistic labels in inductive generalization. Journal of Experimental Child Psychology, 114, 432–455.

Deng, W., & Sloutsky, V. M. (2015). The development of categorization: Effects of classification and inference training on category representation. *Developmental Psychology, 51,* 392-405.

Deng, W., & Sloutsky, V. M. (2016). Selective attention, diffused attention, and the development of categorization. *Clinical Psychology, 91,* 24-62.

Department of Education (1997). *Mathematics equals opportunity.* White Paper prepared for US Secretary of Education Richard W. Riley.

Ferry, A.L., Hespos, S.J., & Waxman, S.R. (2010). Categorization in 3- and 4-month-old infants: An advantage for words over tones. *Child Development, 81*(2), 472-479.

Fulkerson, A.L., & Waxman, S.R. (2007). Words (but not tones) facilitate object categorization: Evidence from 6- and 12-month-olds. *Cognition, 105*(1), 218-228.

Fyfe, McNeil, and Borjas (2015). Benefits of "concreteness fading" for children's mathematics understanding. Learning and Instruction, 35, 104-120.

Fyfe, McNeil, Son, and Goldstone (2014). Concreteness fading in mathematics and science instruction: a systematic review. Educational Psychology Review, 26(1), 9-25.

Geraghty, K., Waxman, S. R., & Gelman, S. (2014). Learning words from pictures: 15- and 17-month-old infants appreciate the referential and symbolic links among words, pictures, and objects. *Cognitive Development, 32*, 1-11.

Goldstone and Son (2005). The transfer of scientific principles using concrete and idealized simulations. The Journal of the Learning Sciences, 14, 69-110.

Halberda, J. (2003). The development of a word-learning strategy. *Cognition, 87*, B23-B34

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature, 455*(7213), 665-668

Kaminski, J. A., & Sloutsky, V. M. (2013). Extraneous perceptual information interferes with children's acquisition of mathematical knowledge. *Journal of Educational Psychology, 105*, 351-363.

Kaminski, J. A., & Sloutsky, V. M. (2009). The effect of concreteness on children's ability to detect common proportion. In N. Taatgen & H. van Rijn (Eds.), Proceedings of the conference of the cognitive science society (pp. 335–340). Mahwah: Erlbaum.

Kaminski, J. A., Sloutsky, V. M., & Heckler, A. F. (2008). The advantage of abstract examples in learning math. *Science*, 320, 454–455. doi:10.1126/science.1154659.

Kaminski, J. A., Sloutsky, V.M., & Heckler, A. F. (2009). Transfer of mathematical knowledge:

the portability of generic instantiations. *Child Development Perspectives*, 3, 151–155.

Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review*, *38*201-221. doi:10.1016/j.dr.2015.07.008

Martin, T., & Schwartz, D. L. (2005). Physically distributed learning: Adapting and reinterpreting physical environments in the development of fraction concepts. *Cognitive Science,* 29, 587-625.

McNeil and Fyfe (2012). “Concreteness fading” promotes transfer of mathematical knowledge. Learning and Instruction, 22, 440-448.

McNeil and Uttal (2009). Rethinking the use of concrete materials in learning: Perspectives from development and education. Child Development Perspectives, 3(3), 137-139.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA.

Peterson, L.A., & McNeil, N.M. (2012). Effects of Perceptually Rich Manipulatives on Preschoolers’ Counting Performance: Established Knowledge Counts. *Child Development, 84*(3), 1020-1033.

Plebanek, D. J., & Sloutsky, V. M. (2017). Costs of selective attention: when children notice what adults miss. *Psychological science,* doi: 0956797617693005.

Posid, T., & Cordes, S. (2014). Verbal counting moderates perceptual biases found in children’s cardinality judgments. *Journal of Cognition and Development, 16*(4), 621-637

Posid, T., & Cordes, S. (2017). How high can you count? Probing the limits of children’s counting. *Developmental Psychology.*

Posid, T., & Sloutsky, V. M. (2017). *When less isn’t more: A real-world fraction intervention study.* Poster presented at the biennial meeting of the Society for Research in Child

Development. Austin, Texas.

Posid, T., & Sloutsky, V. M. (2016). *Kindergarteners and adults learn fraction rules in a categorization task*. Poster presented at the annual meeting of the Cognitive Science Society. Philadelphia, Pennsylvania.

Posid, T., Mills, A. K., & Sloutsky, V. M. (in preparation). When less is more: Perceptual features count under difficult task demands.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., et al. (2012). Early predictors of high school mathematics achievement. Psychological Science, 23, 691–697. doi:10.1177/0956797612440101.

Siegler, R. S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. Child Development Perspectives, 8, 144–150. doi:10.1111/cdep.12077.

Son, J. Y., Smith, L. B., & Goldstone, R. L. (2011). Connecting instances to promote children's relational reasoning. *Journal of Experimental Child Psychology,* 108(2), 260-277.

Son, J. Y., Smith, L. B., & Goldstone, R. L. (2008). Simplicity and generalization: Short-cutting abstraction in children's object categorizations. *Cognition, 108*(3), 626-638. doi:10.1016/j.cognition.2008.05.002

Waxman, S. R., & Braun, I. (2005). Consistent (but not variable) names as invitations to form object categories: New evidence from 12-month-old infants. *Cognition*, 95(3), B59–B68.

Waxman, S. R., & Markow, D. B. (1995). Words as invitations to form categories: Evidence from 12- to 13-month-old infants. Cognitive Psychology, 29, 257–302.

Table 1

Average performance on each Condition of Abstraction Task in Experiments 1 and 2 by Trial Type

	Trial Type				
Condition	1(BW circle)	2(color circle)	3(BW non-circle)	4(Color non-circle)	5(Rich)
One [3/4]	81.3%	76.9%	67.5%	83.8%	73.8%
Multiple [3/4]	66.7%	66.7%	53.3%	80.0%	60.0%
Multiple [3/4]	66.7%	62.2%%	46.7%	57.8%	42.2%
Dax [3/4]	77.6%	81.6%	69.7%	89.5%	77.6%
Three [3/4]	88.1%	89.9%	66.7%	92.9%	79.8%

Table 2

Condition	*Pre-test*	*Post-test*
One	36.7%	37.6%
Multiple	40.0%	48.3%
Dax	31.0%	29.2%
Three	32.3%	41.4%

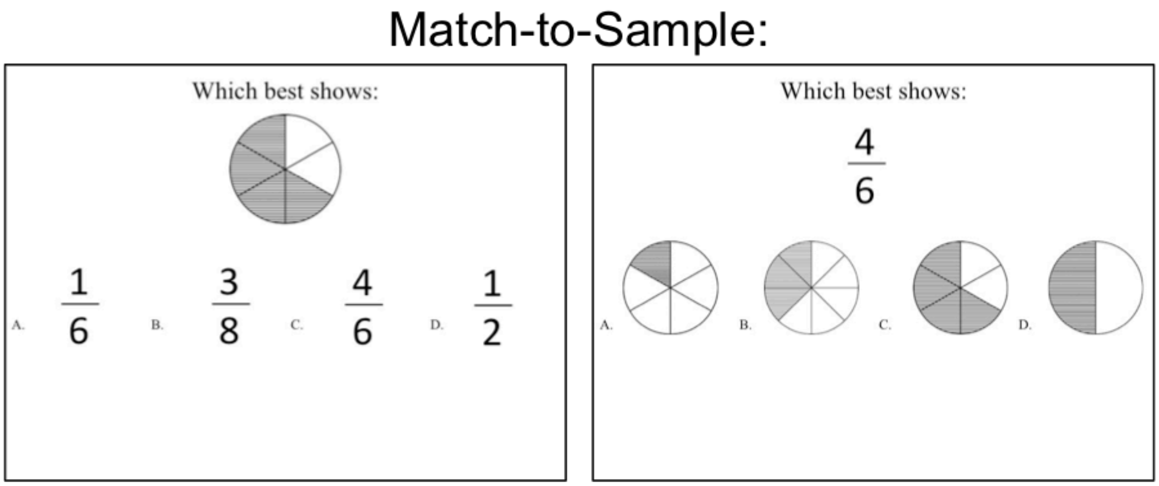

Figure 1. Fraction matching task used in pre-test and post-test.

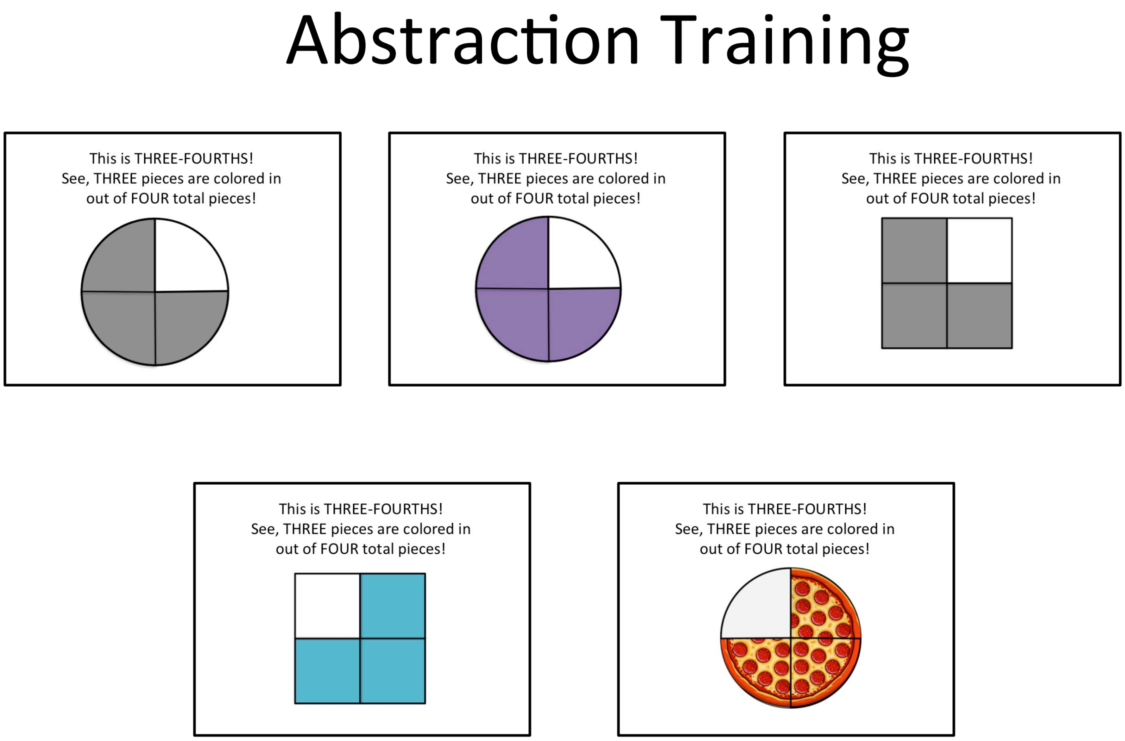

Figure 2. Abstraction task training stimuli.

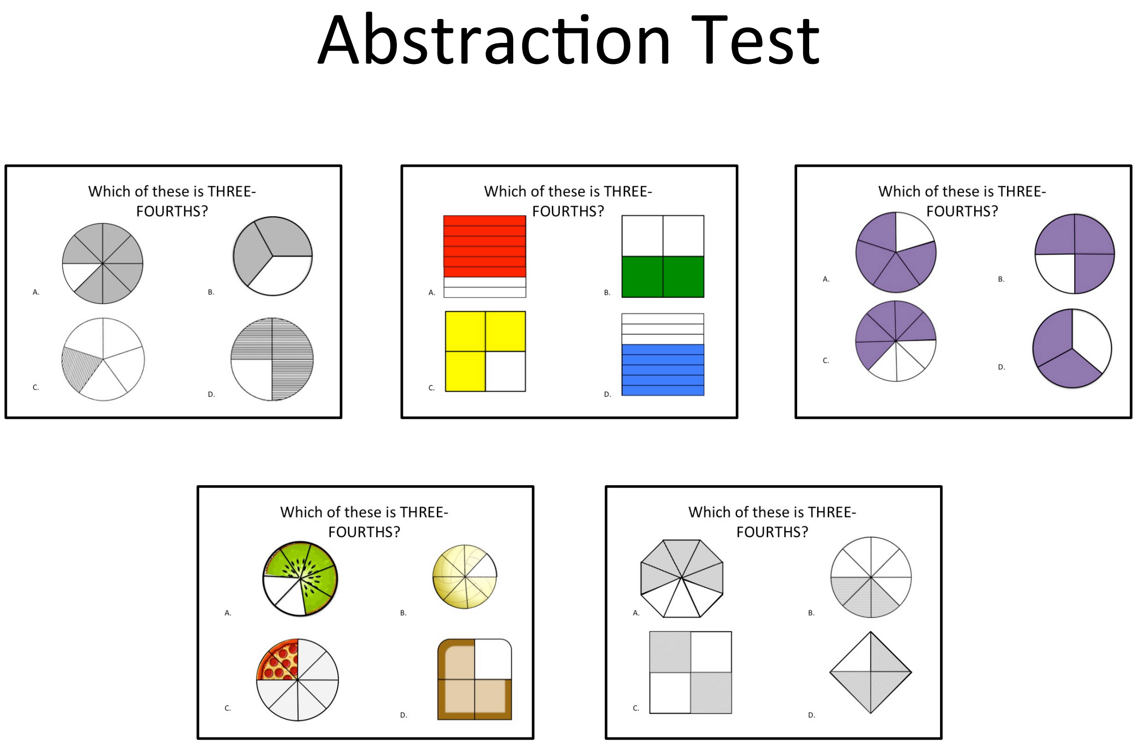

Figure 3. Abstraction task test stimuli.

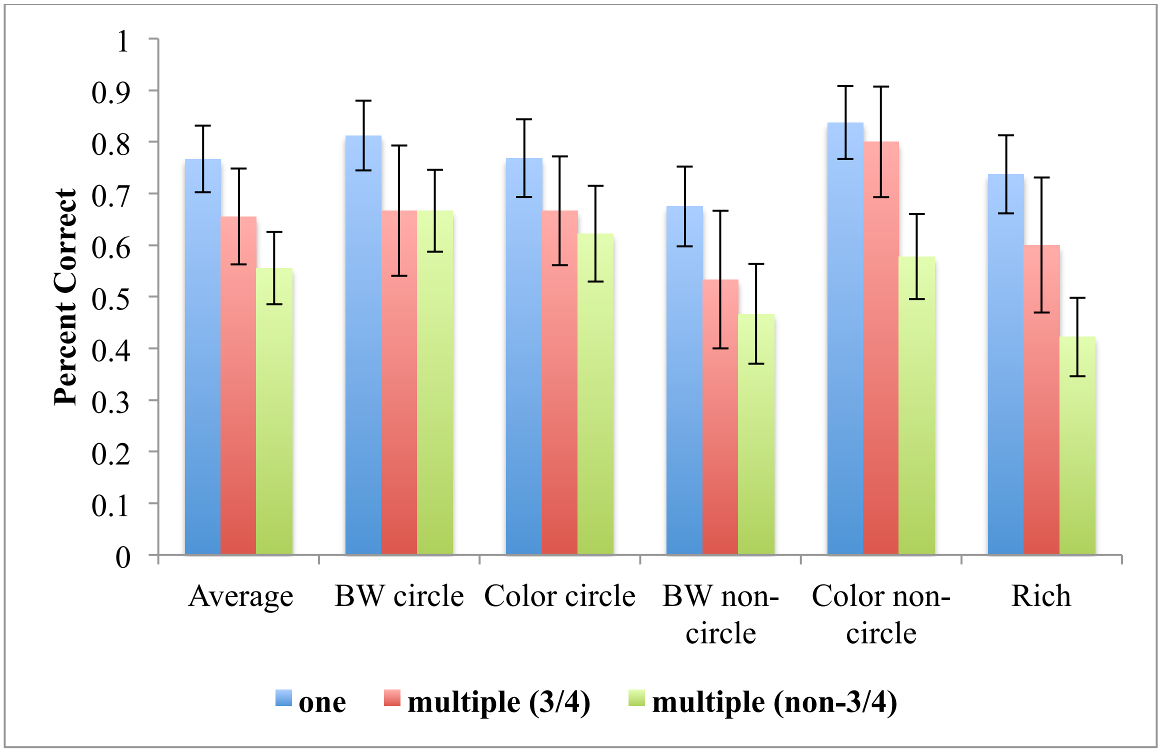

Figure 4. Children performed >chance on the Single Exemplar condition, which held across all trial types (all *p's*<.001; *Cohen's d's*>2.5)**.** In the Multiple Exemplar condition, children also performed above chance on ¾ (all *p's*<.053, *Cohen's d's* > 1.14) and on non-3/4 fractions (all *p's*<.05, *Cohen's d's*>1.2). Error bars represent Standard Error of the Mean.

.

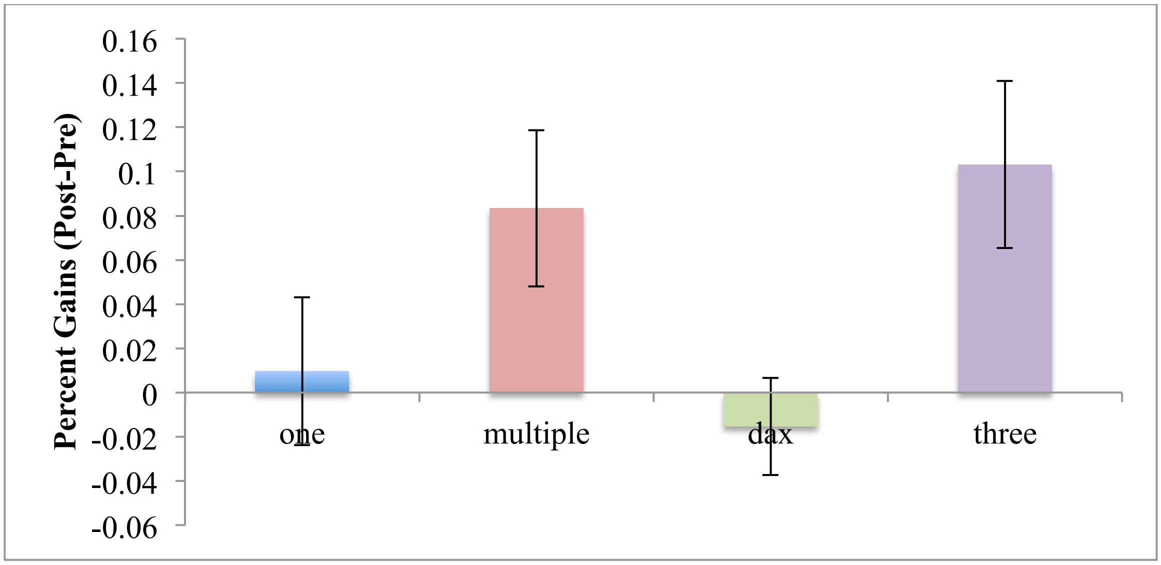

Figure 5. Children demonstrated significant pre-post test gains in the Multiple and Three Exemplar Conditions (*p's*<.05 vs. chance), but not in the Single and Dax Exemplar Conditions (*p*>.5 vs. chance). Error bars represent Standard Error of the Mean.

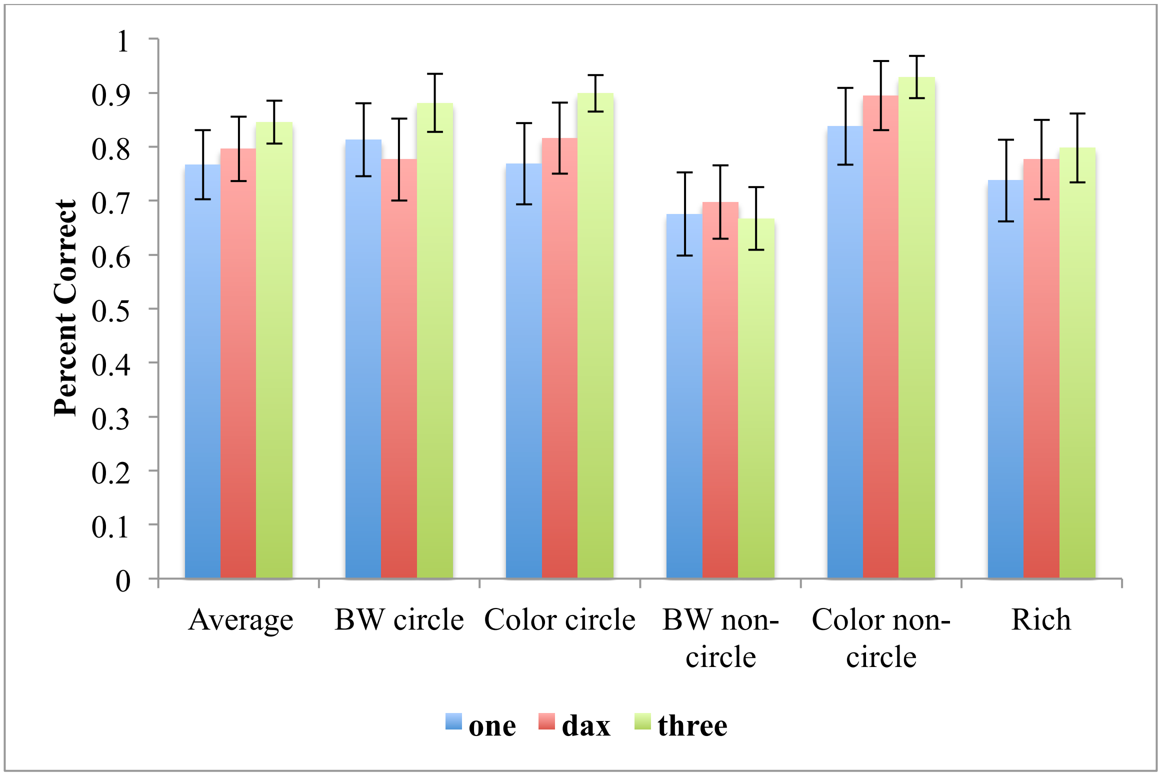

Figure 6. Children performed >chance overall and across the 5 trial types in both the Dax and Three conditions (*p's*<.001). Single vs. Dax : accuracy on abstraction task (*p*>.7), accuracy across trials (all *p's*>.5), and different score gains (*p*>.5). Single vs. Three : accuracy on abstraction task (*p*>.3), accuracy across trial types (all *p's*>.1), but difference in gains (*p*=.073, *Cohen's d*=0.59). Error bars represent Standard Error of the Mean.